June, 2024

Wrinkling of Liquid-infused Membranes

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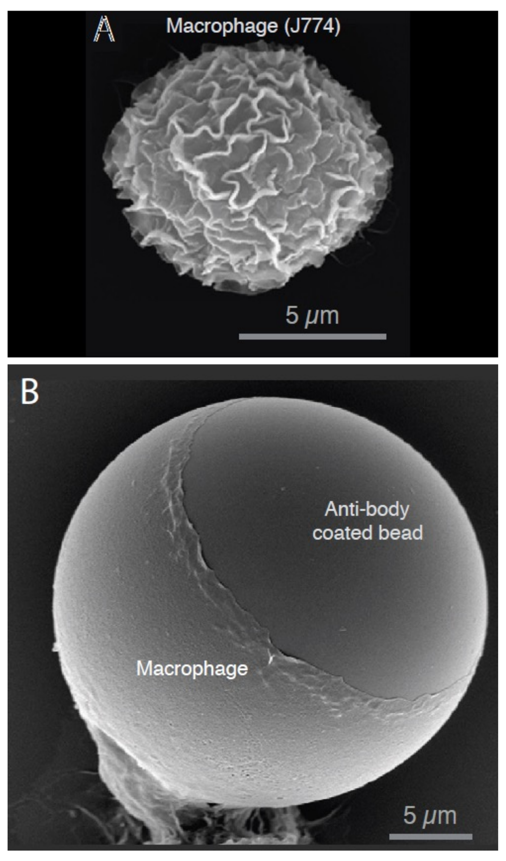
Context

- Deformation of a synthetic elastic membrane under capillary forces
- Analogy : deformation of biological tissues
- Applications : stretchable electronics, smart textiles, soft biomedical devices…

Key words

- Elastocapillarity
- Thin structures
- Wrinkling
- Surface tension
- Phase transformation



Lam et al. Biophys. J (2009)

Deformation of a macrophage

Experiment

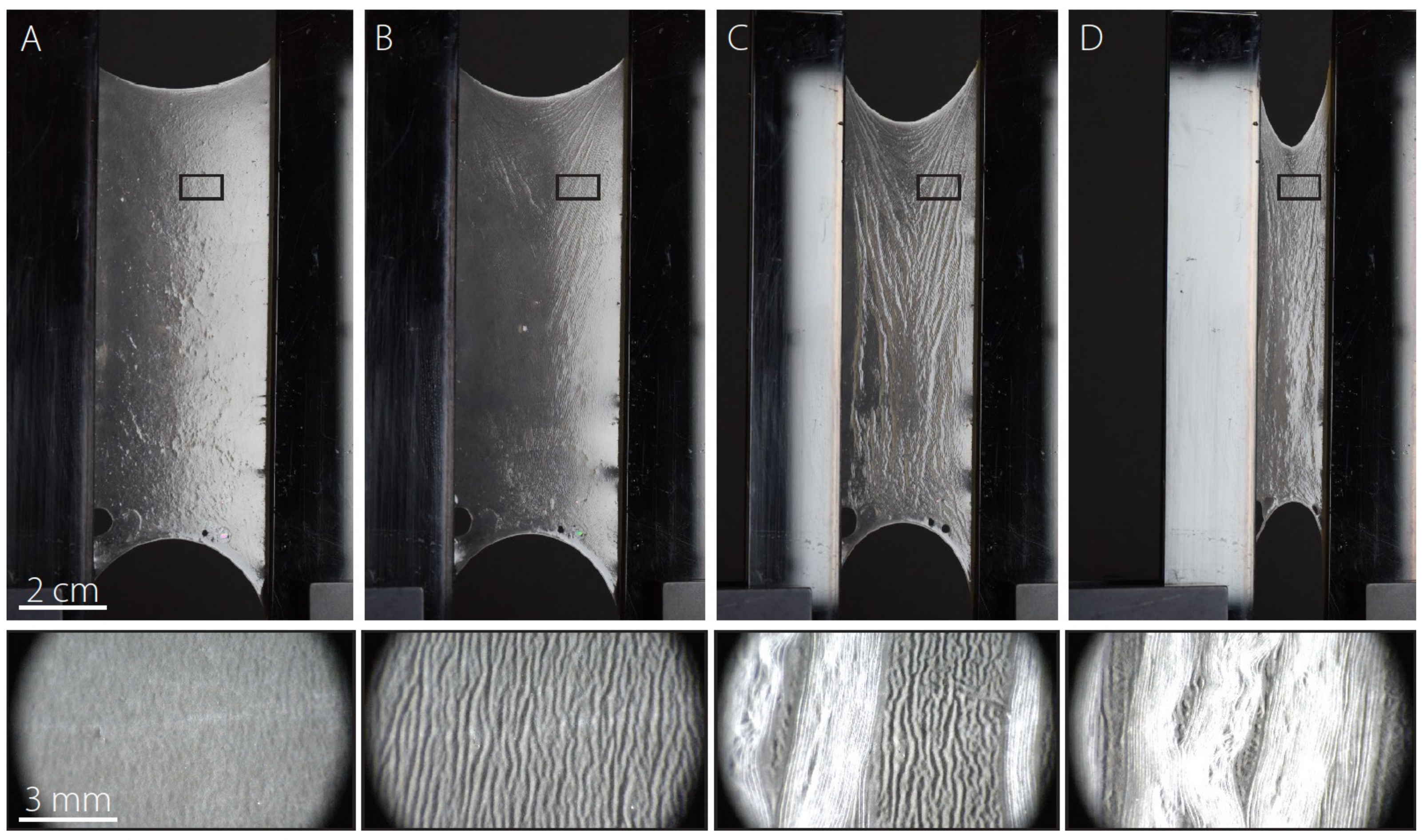
(Top view)

The membrane is clamped at both ends.

Observation :

- Small displacement : homogeneous wrinkles.
- Large displacement : Two zones, the percentage of which changes with the displacement

How do wrinkles form? Phase transformation?



P.Grandgeorge, PhD thesis, (2018)

Modeling

Basic idea : Energy minimization

Total energy :

$$\mathcal{E}_{tot} = \mathcal{E}_{elas} + \mathcal{E}_{\gamma}$$

$\mathcal{E}_{elas}$: Elastic bending energy

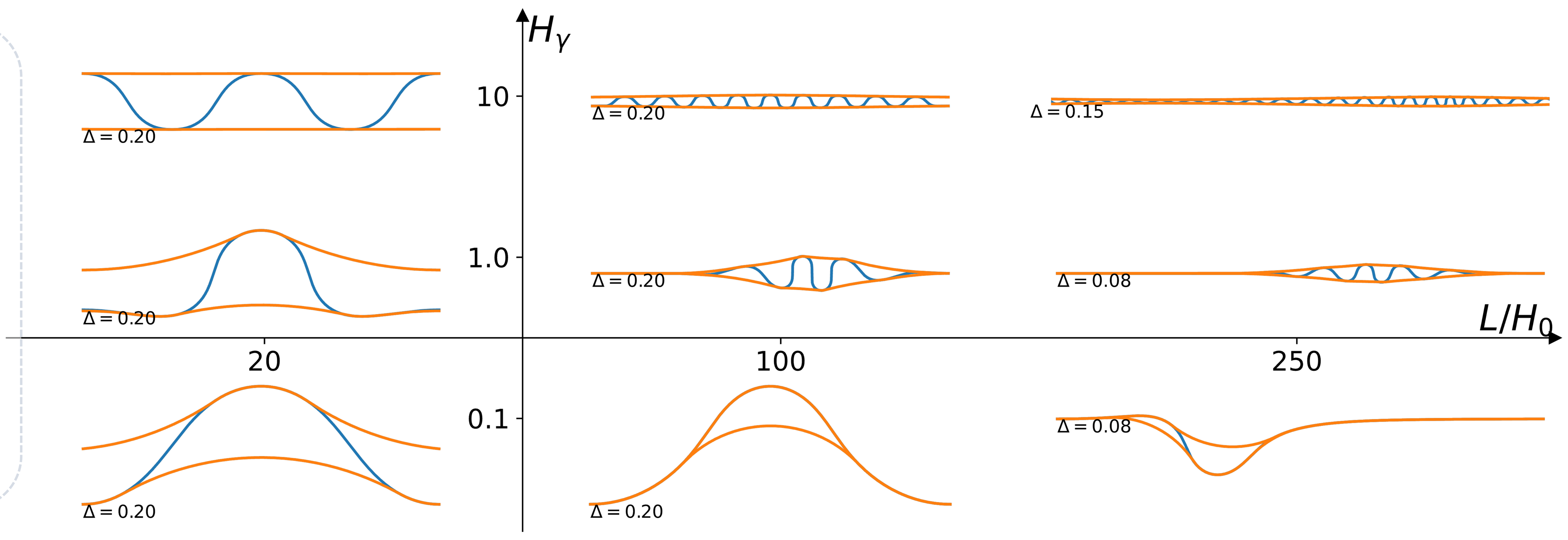
$\mathcal{E}_{\gamma}$: Surface energy

Hypothesis :

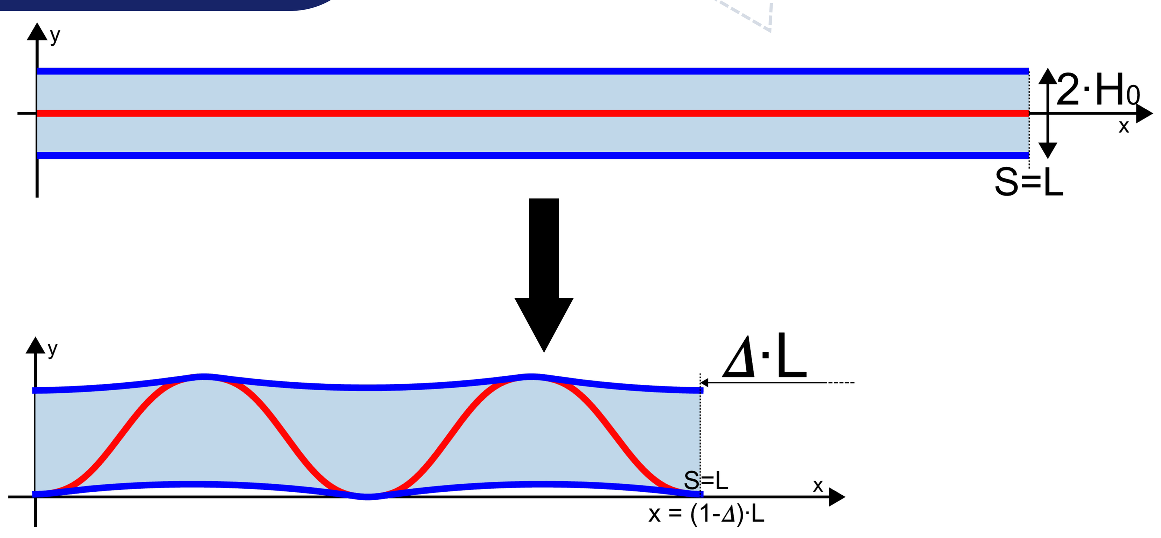
- Inextensible membrane
- No interface penetration
- Liquid volume keeps constant

The problem is solved as an optimization problem, discretized and implemented in a python optimizer

CasADi



Whole system simulation



Parameters :

- End shortening : Δ
- Initial liquid thickness : H_0
- Total length of the membrane : L

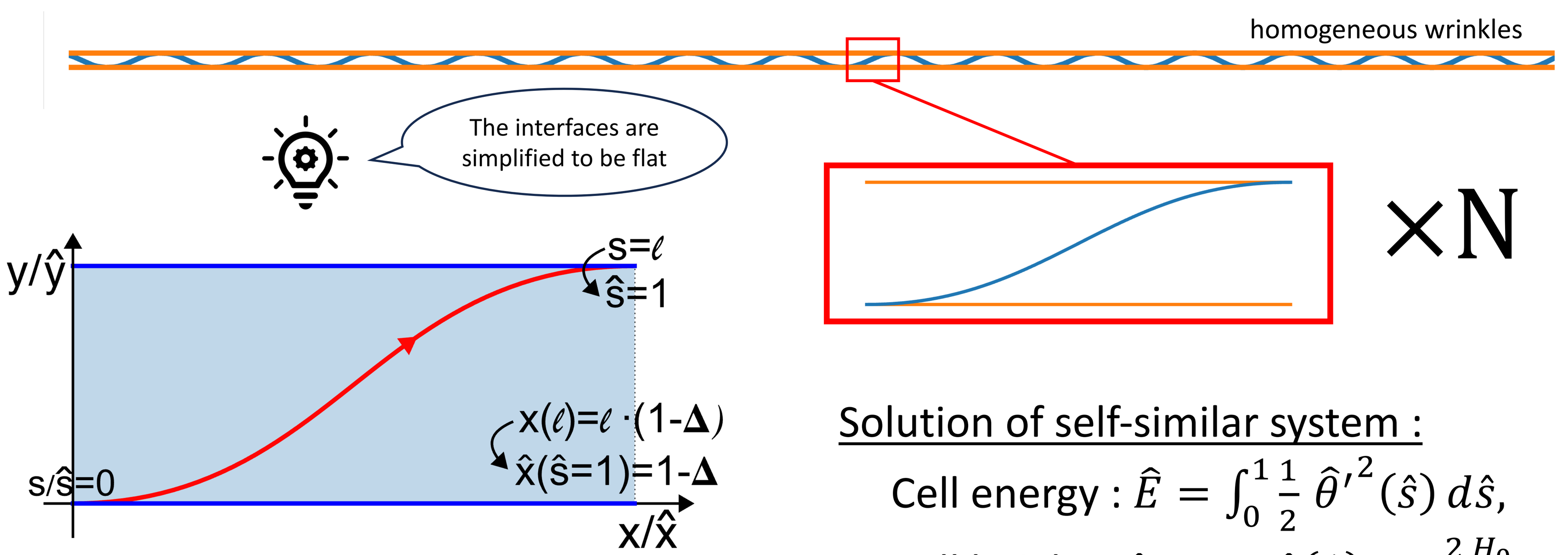
Properties :

- Bending stiffness : $D = \frac{1}{12}Et^3$
- surface tension : γ

Dimensional analysis :

- Elasto-capillary length $\sqrt{\frac{D}{\gamma}}$
- Magnitude of capillary effects $H_\gamma = \sqrt{\frac{\gamma}{D}}H_0$
- Membrane length vs. liquid volume L/H_0

Cell model



Unknown & Variables :

$$l\,,\quad \theta(s),\quad x(s),\quad y(s)$$

Self-similar variables :

$$\hat{s} = \frac{s}{l}, \hat{x} = \frac{x}{l}, \hat{y} = \frac{y}{l},$$

$$\hat{n}_x = \frac{n_x}{l^2}, \hat{n}_y = \frac{n_y}{l^2}$$

Self-similar system :

$$\hat{\theta}'(\hat{s}) = \kappa(\hat{s})$$

$$\hat{x}'(\hat{s}) = \cos\hat{\theta}(\hat{s})$$

$$\hat{y}'(\hat{s}) = \sin\hat{\theta}(\hat{s})$$

$$\hat{\theta}''(\hat{s}) = \hat{n}_x \sin\hat{\theta}(\hat{s}) - \hat{n}_y \cos\hat{\theta}(\hat{s})$$

$$\frac{1}{2}\hat{\theta}'^2(1) + \hat{n}_x \cdot \Delta = \frac{\hat{n}_y}{2}\hat{y}(1)$$

Solution of self-similar system :

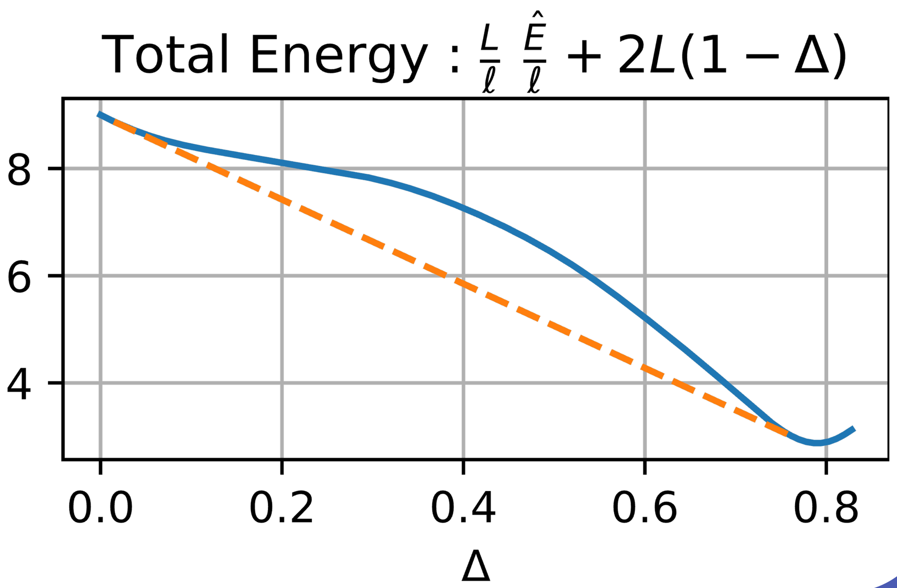
Cell energy : $\hat{E} = \int_0^1 \frac{1}{2}\hat{\theta}'^2(\hat{s})\,d\hat{s}$,

Cell height : $\hat{y}_{end} = \hat{y}(1) = \frac{2H_0}{(1-\Delta)l}$

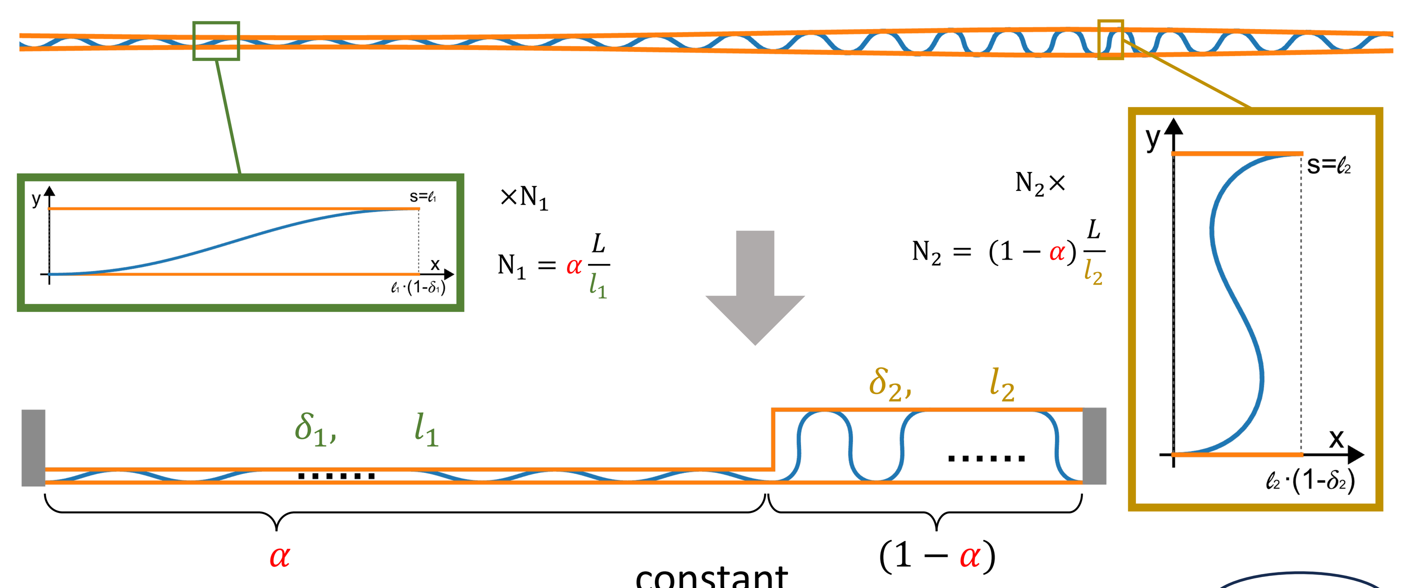
which are functions of Δ

Finding the variable length l :

$$l = \frac{2H_0}{(1-\Delta)\cdot\hat{y}_{end}(\Delta)}$$



Two phases cell model



Energy :

$$\mathcal{E}_{tot} = N_1E_1 + N_2E_2 + 2L(1-\Delta)$$

constant

Objective :

$$\min_{(\alpha, l_i, \delta_i)} \alpha\frac{L}{l_1}\frac{\hat{E}(\delta_1)}{l_1} + (1-\alpha)\frac{L}{l_2}\frac{\hat{E}(\delta_2)}{l_2}$$

Constraints :

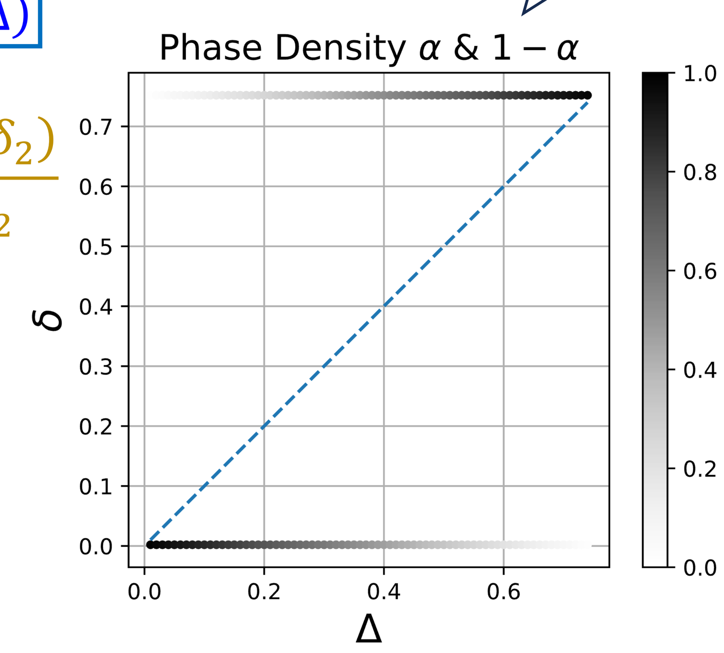
Displacement control :

$$\alpha\delta_1 + (1-\alpha)\delta_2 = \Delta$$

Fixed volume :

$$\alpha h_1 + (1-\alpha)h_2 = H_0$$

$$h_i = l_i \cdot \hat{y}_{end}(\delta_i)(1-\delta_i)$$



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