Surface wrinkling of a thin liquid-infused membrane

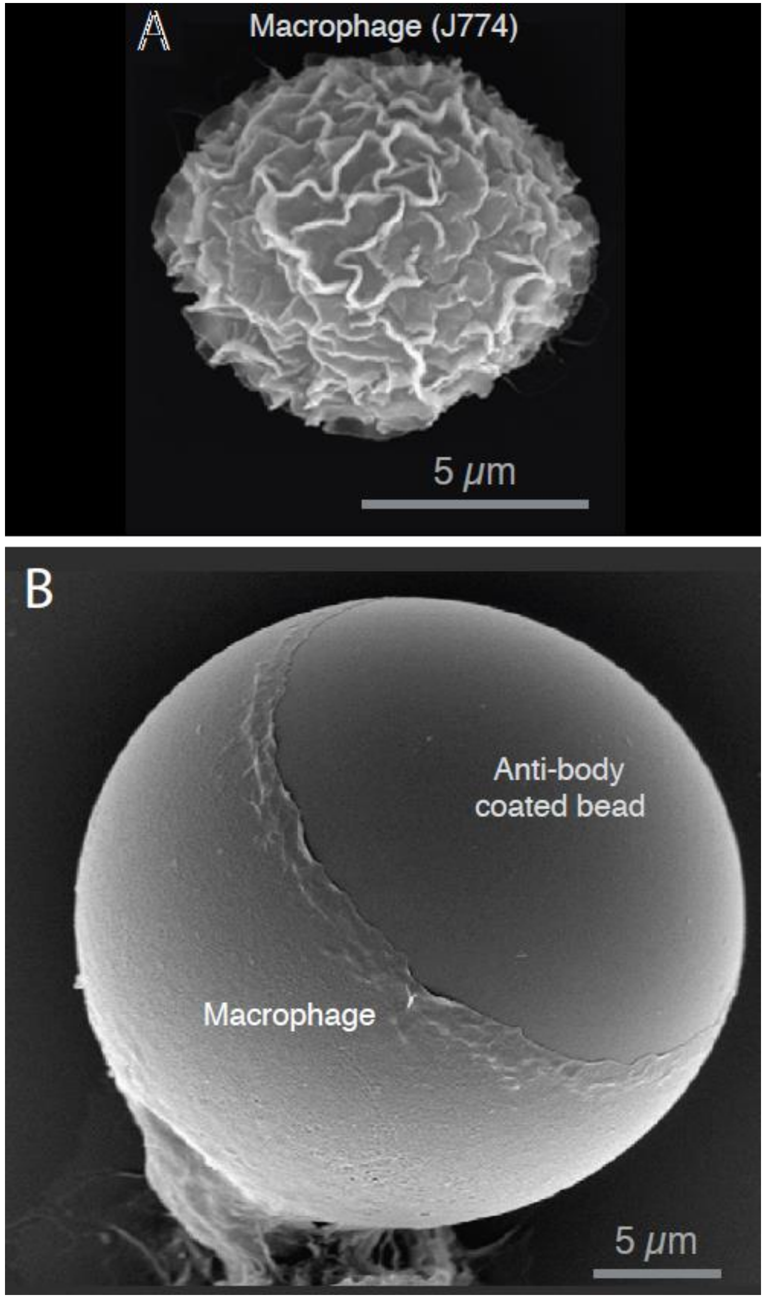
Jiayu Wang & Sébastien Neukirch & Arnaud Antkowiak

∂'Alembert Institute, Sorbonne Université, CNRS UMR 7190, Paris - France

Introduction & Motivation : Surface reservoirs

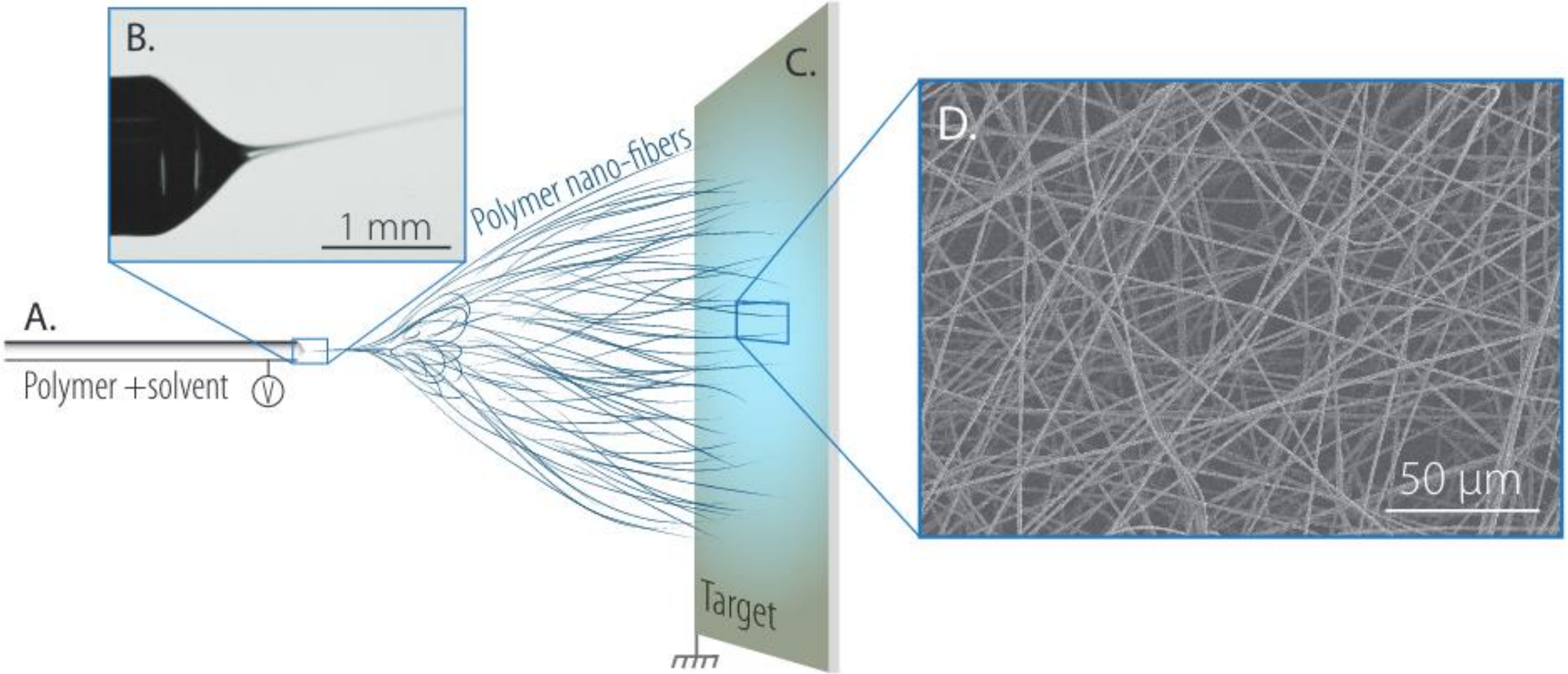
- Surface reservoirs in nature :
  Cytomembrane of macrophage

- Reproduce surface reservoir by elasto-capillarity ?



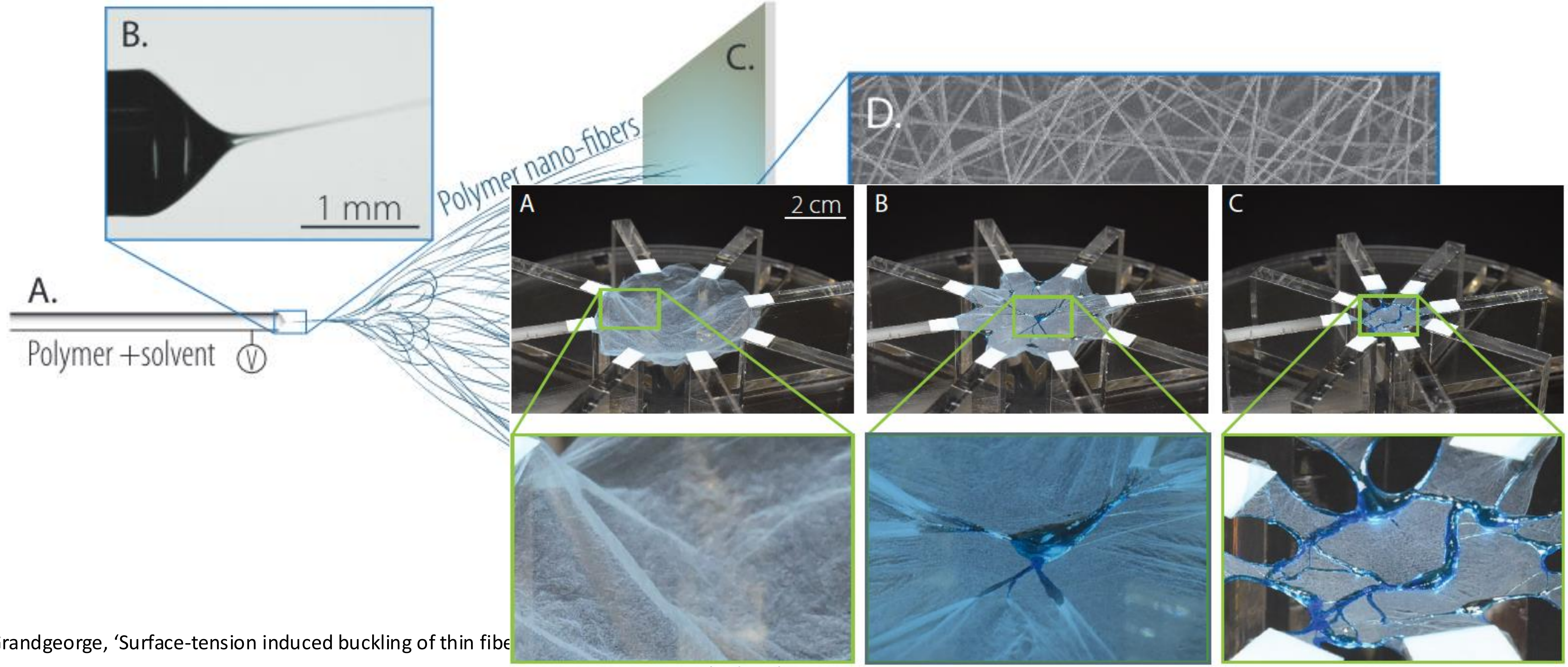
J. Lam, M. Herant, M. Dembo, and V. Heinrich. Baseline mechanical characterization of J774 macrophages. Biophysical Journal, 96(1):248–254, 2009.

Fabrication of the fibrous membrane



P. Grandgeorge, ‘Surface-tension induced buckling of thin fibers and fibrous membranes: a novel strategy to design stretchable materials’, phd thesis, Sorbonne Université, 2018.

Fabrication of the fibrous membrane



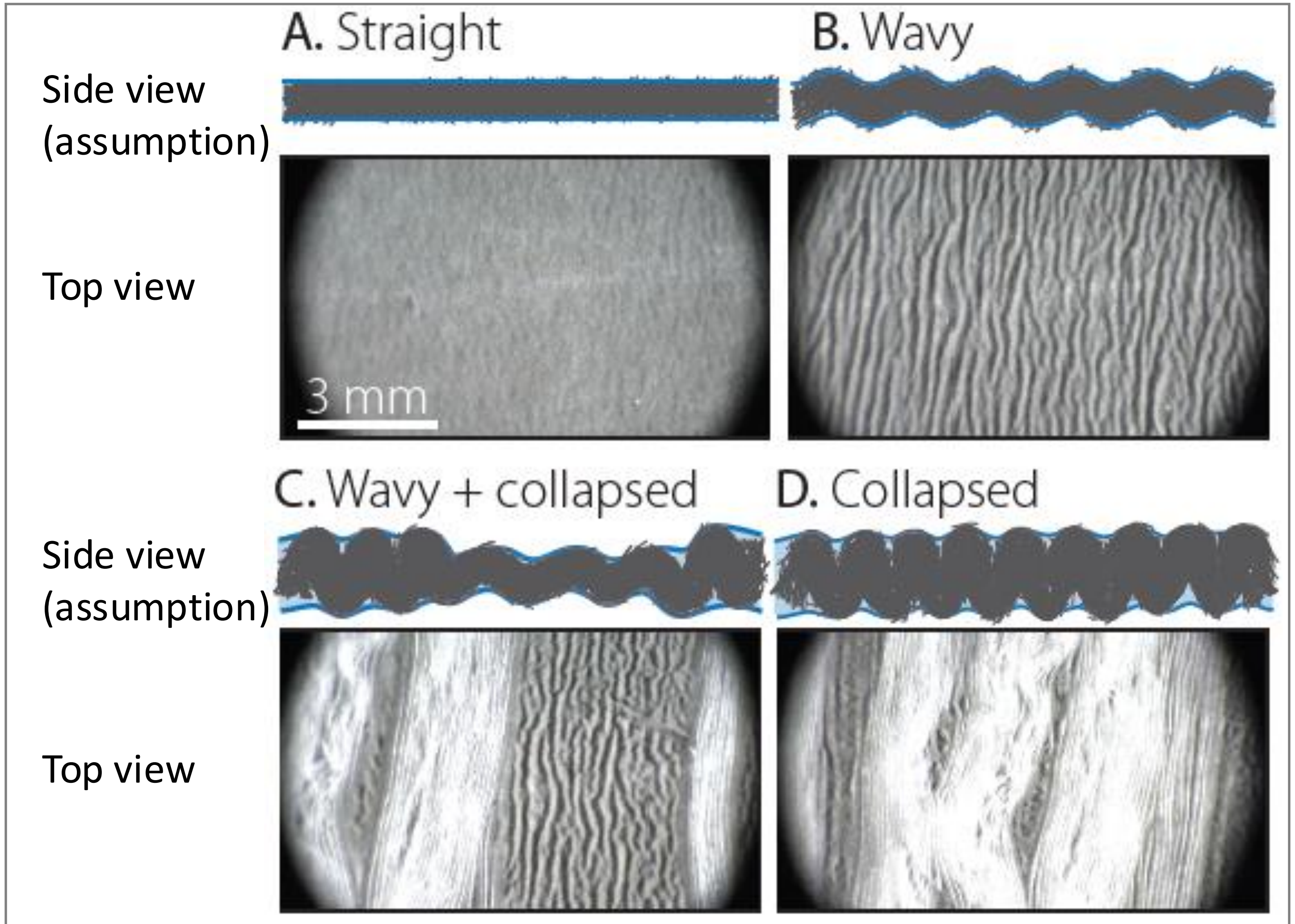
P. Grandgeorge, 'Surface-tension induced buckling of thin fibe

A glance at the wrinkling

<u>Materials :</u>

- Membrane :
  PVDF-HFP
- Liquid :
  silicon oil

One-directional compression

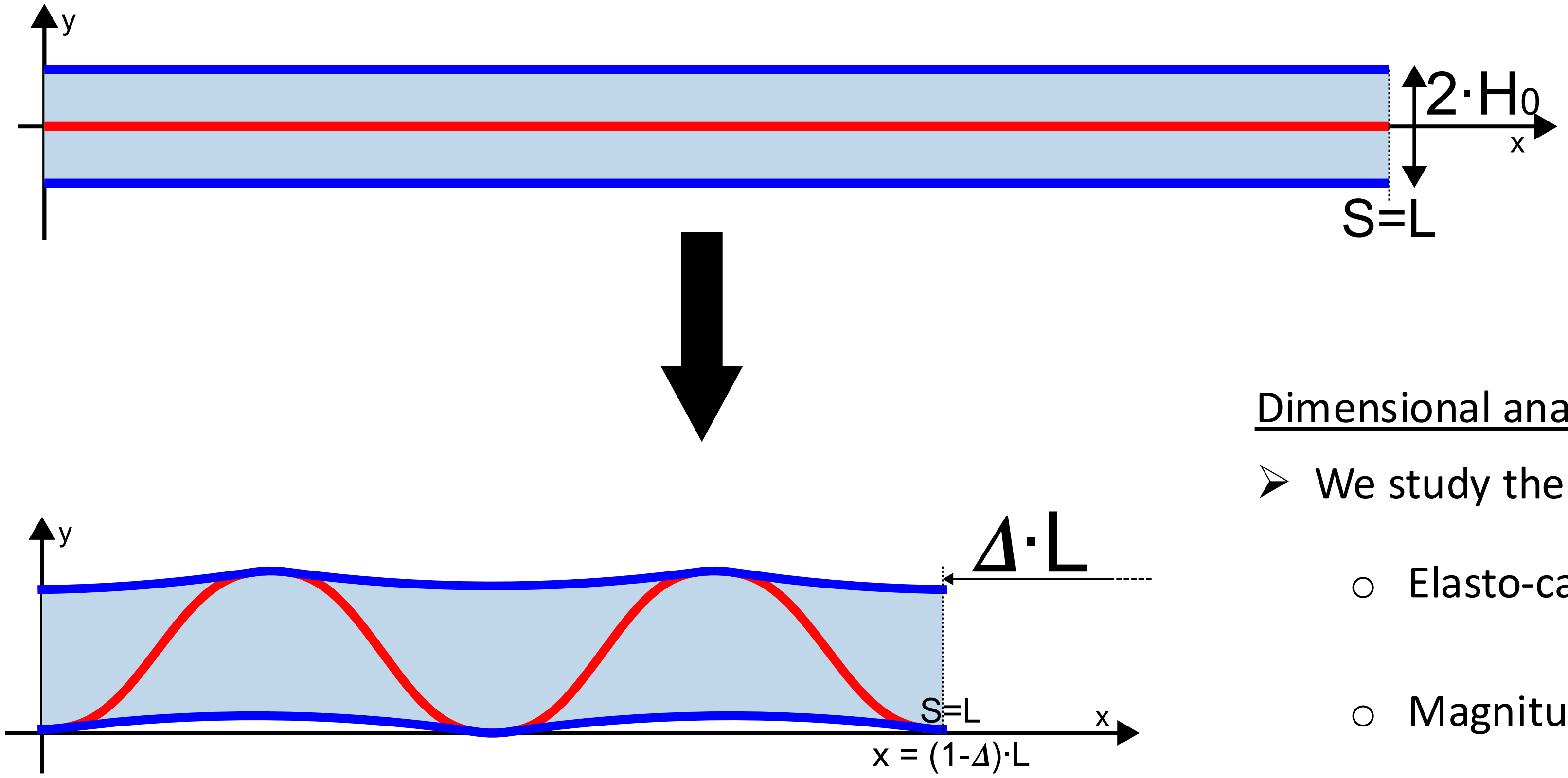


P.Grandgeorge, thesis, (2018)

Modelization

A model explaining wavy pattern **and** the co-existing phases ?

Modelization : membrane + interfaces



Parameters :

Δ (End shortening)

H_0 (Volume related)

L (Total length)

Properties :

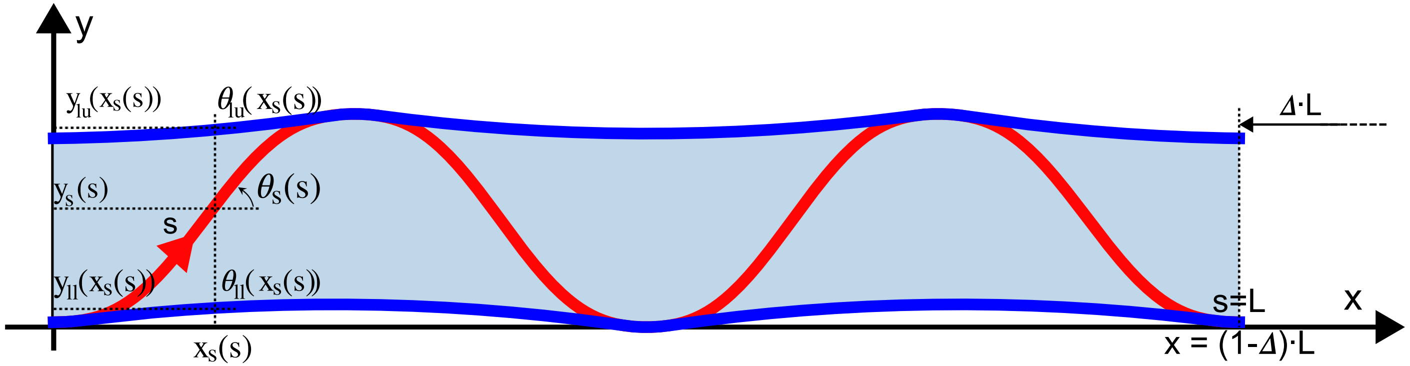
$D = \frac{1}{12} E t^3$ (Bending stiffness)

γ (surface tension)

Dimensional analysis :

- We study the system behaviors with
  - Elasto-capillary length $\sqrt{\frac{D}{\gamma}}$
  - Magnitude of capillary effects $H_\gamma = \sqrt{\frac{\gamma}{D}} H_0$
  - Membrane length vs. liquid volume L/H_0

System model : energy minimization



Energy :

$$\mathcal{E}_{elas} = \int_{s=0}^{l} \frac{Dw}{2} \kappa_s{}^2(s)\, ds$$

$$\mathcal{E}_\gamma = \gamma w (L_{lu} + L_{ll})$$

Objective :

$$\min\ \mathcal{E}_{tot} = \mathcal{E}_{elas} + \mathcal{E}_\gamma$$

Contact constraint :

$$y_{ll}(x_s) \le y(x_s) \le y_{lu}(x_s)$$

Volume conservation :

$$\mathcal{V}_{liquid} = \int_{s=0}^{L} [y_{lu}(x_s) - y_{ll}(x_s)] \cos\theta_s\, ds$$

$$= 2L\, w\, H_0$$

(H_0 : initial liquid thickness at one sides)

Variables:

$\kappa_s(s)$, $\theta_s(s)$, $x_s(s)$, $y_s(s)$

$\theta_{ll}(x_s(s))$, $y_{ll}(x_s(s))$, $\theta_{lu}(x_s(s))$, $y_{lu}(x_s(s))$

The problem is solved as an optimization problem, discretized and implemented to a python optimizer

CasADi

Geometry (2D, width = w):

$$\theta_s{}'(s) = \kappa_s(s),$$
$$x_s{}'(s) = \cos\theta_s(s),$$
$$y_s{}'(s) = \sin\theta_s(s)$$

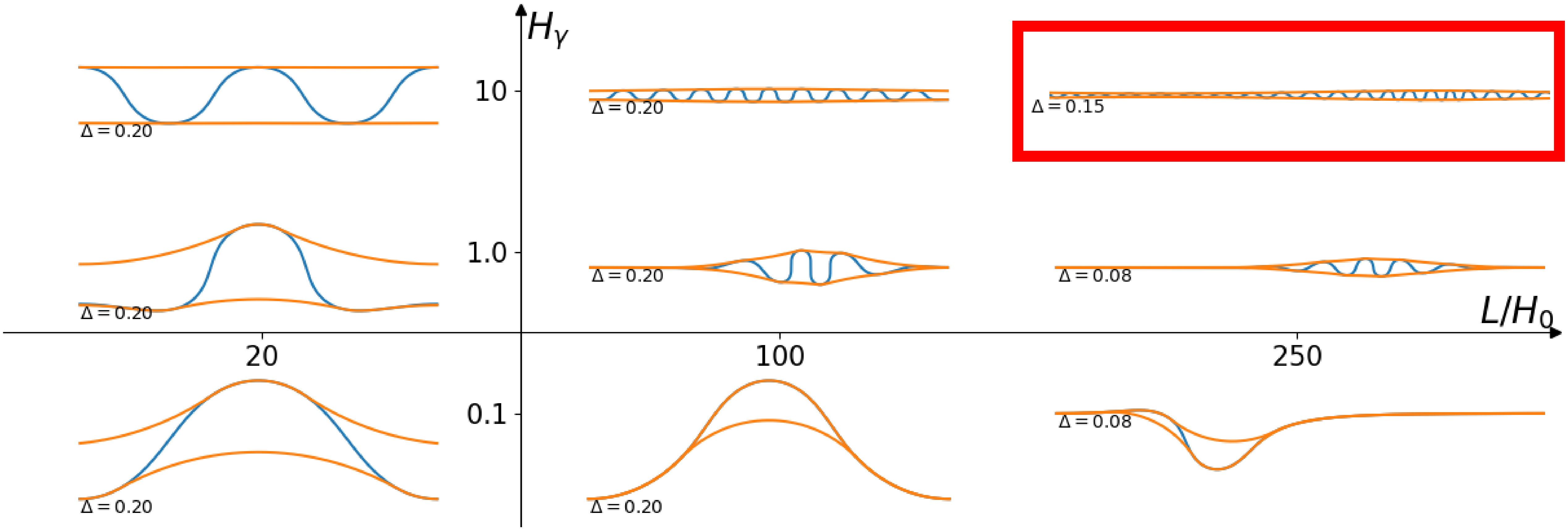
Boundary conditions : Clamped, periodic

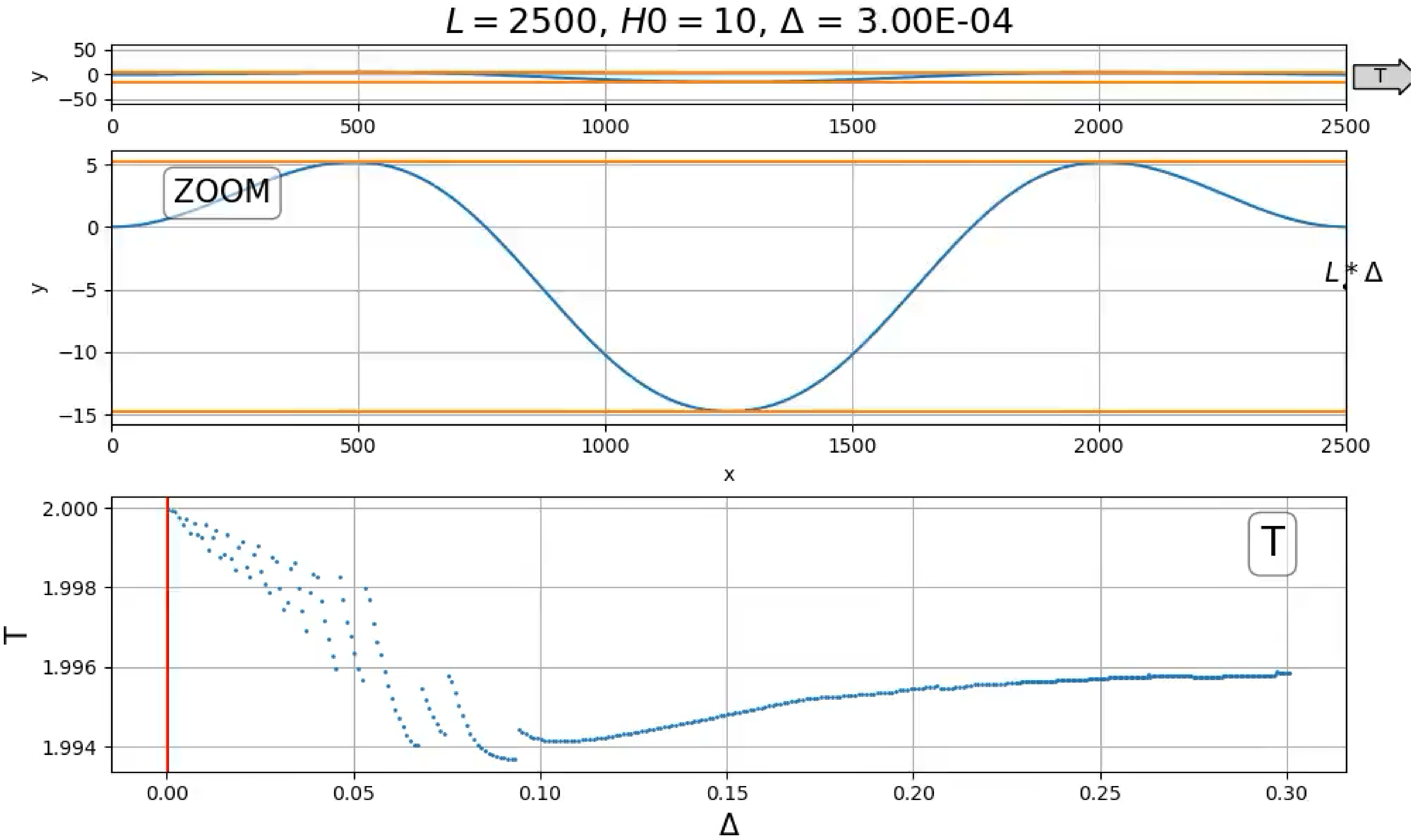
Nondimensionalization with :

$\sqrt{\frac{D}{\gamma}}$ (length scale), γ (force scale)

Parametric Study

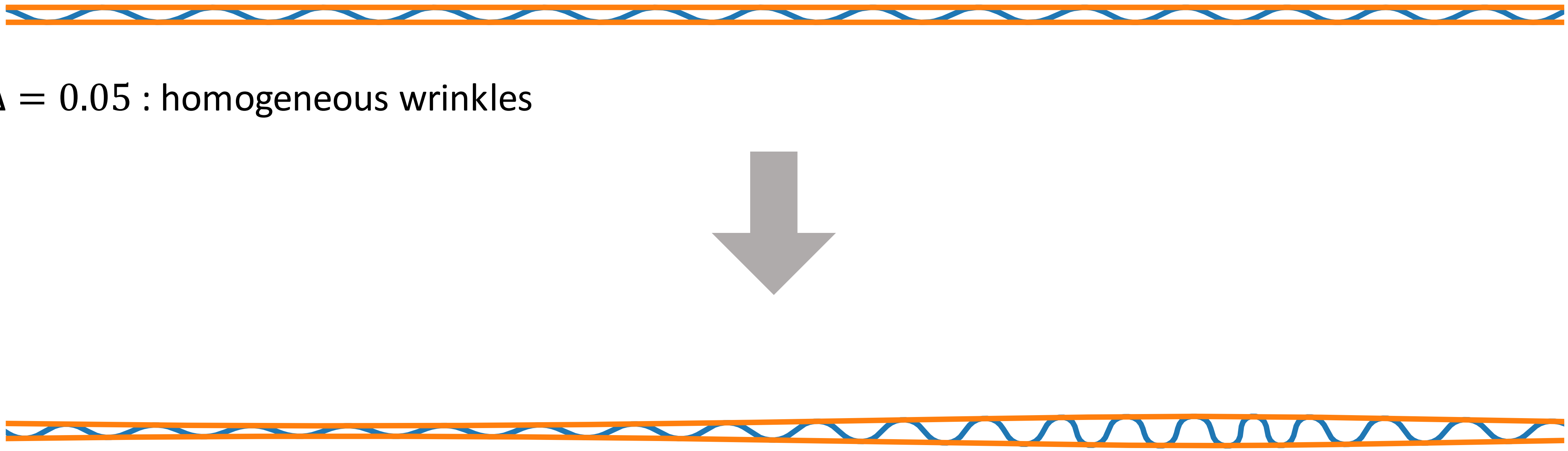
- Elasto-capillary length $\sqrt{\frac{D}{\gamma}}$
- Magnitude of capillary effects $H_\gamma = \sqrt{\frac{\gamma}{D}} H_0$
- Membrane length vs. liquid volume L/H_0





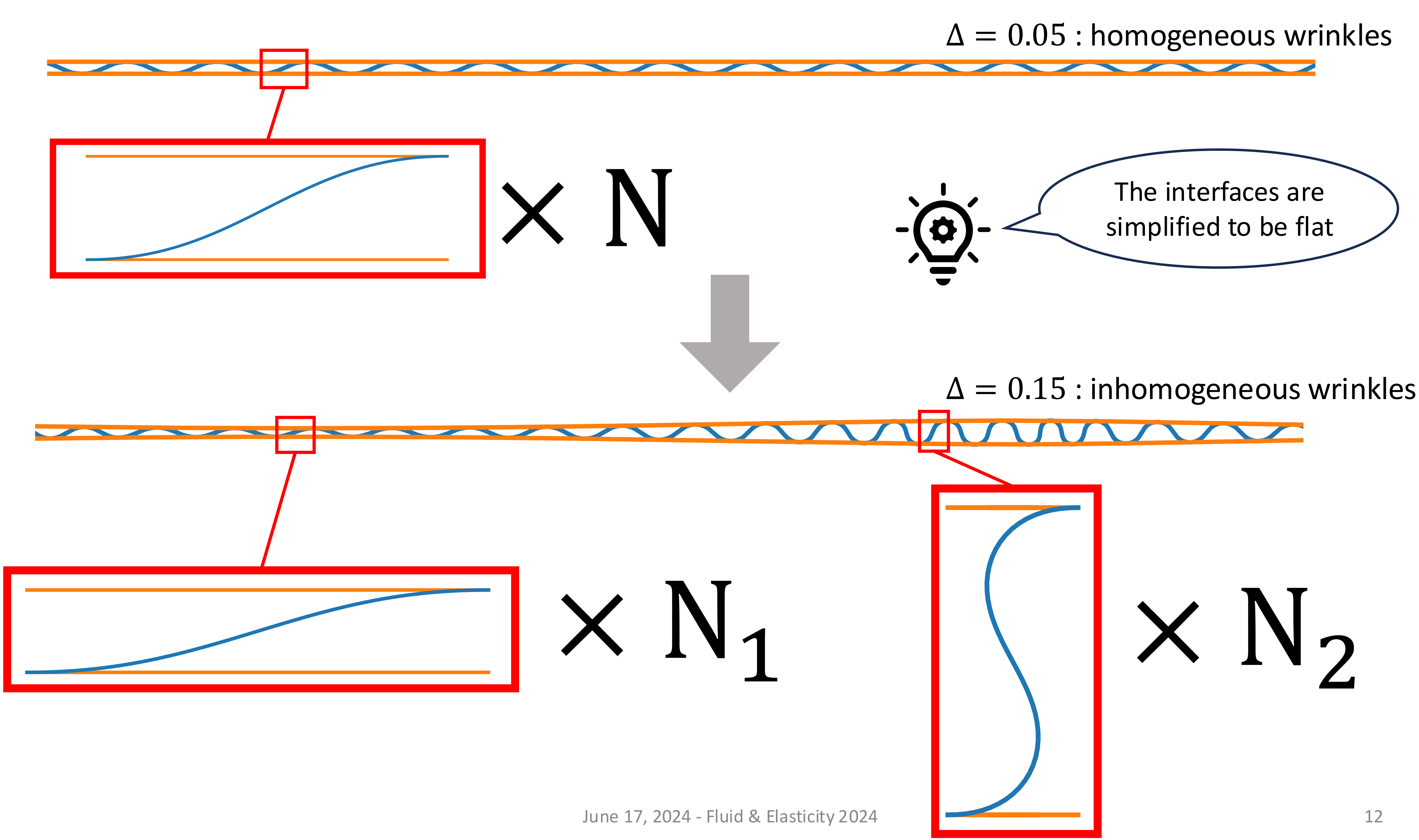
$L = 2500$, $H0 = 10$, Δ = 3.00E-04
T
ZOOM
$L * \Delta$
x
T
Δ

Case : large H_γ and large L/H_0



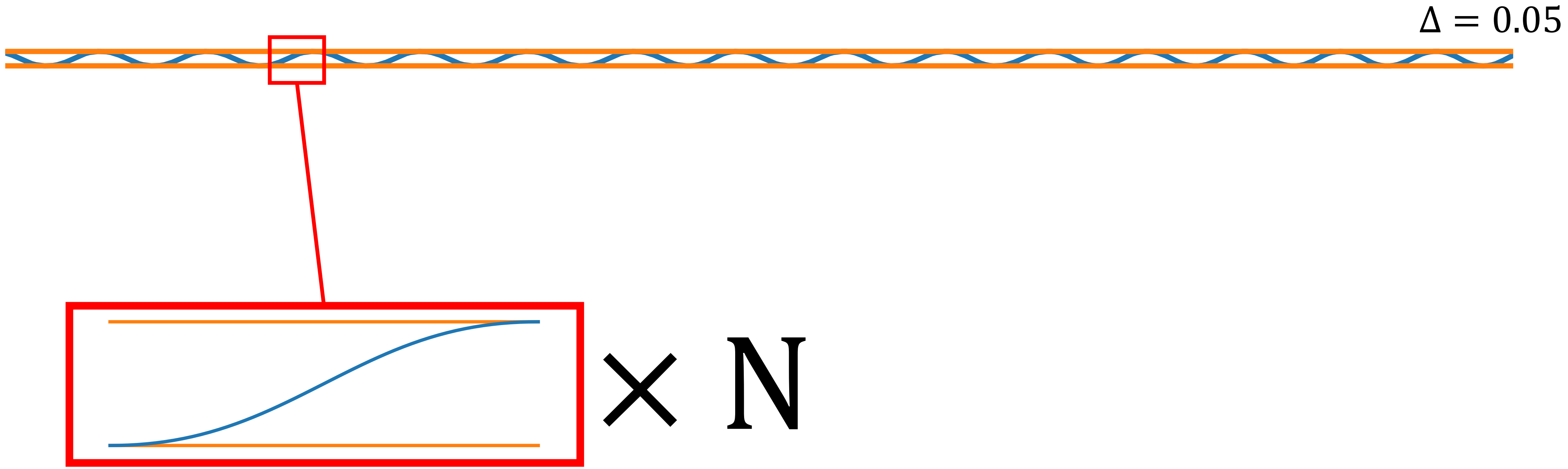
$\Delta = 0.05$: homogeneous wrinkles

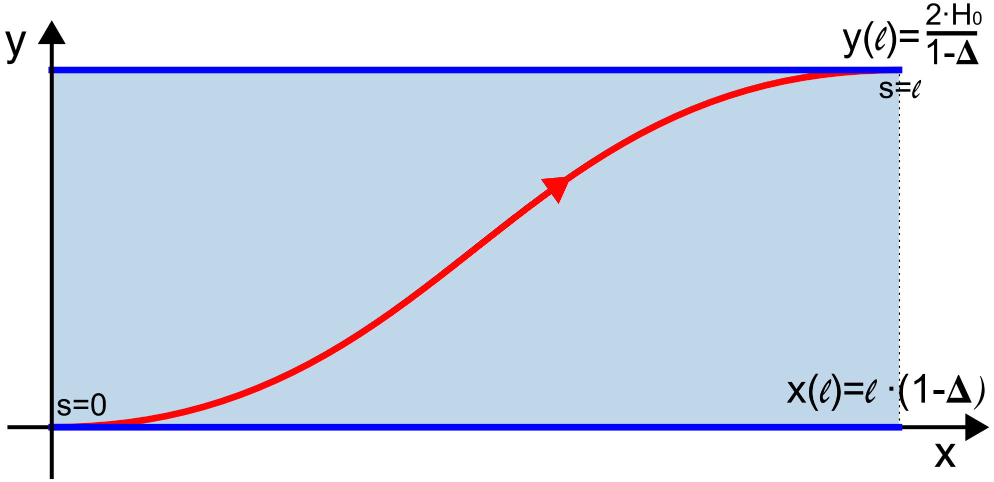
$\Delta = 0.15$: inhomogeneous wrinkles



$\Delta = 0.05$: homogeneous wrinkles
$\times$ N
The interfaces are simplified to be flat
$\Delta = 0.15$: inhomogeneous wrinkles
$\times \mathrm{N}_1$
$\times \mathrm{N}_2$

Homogeneous wrinkles





Energy (dimensionless) :

$$\mathcal{E}_{elas} = \int_{s=0}^{l} \frac{1}{2}\,\theta'^2(s)\,ds$$

$$\mathcal{E}_\gamma = 2\cdot x(\ell)$$

Objective :

$$\min\ \mathcal{E}_{tot} = \frac{L}{l}\left(\mathcal{E}_{elas} + \mathcal{E}_\gamma\right)$$

$$\min\ \mathcal{E}_{tot} = \frac{L}{l}\mathcal{E}_{elas} + 2L(1-\Delta)$$ ⇒ constant

Unknown & Variables :

$\theta(s)$, $x(s)$, $y(s)$, l

Geometry :

$$x'(s) = \cos\theta\,(s),$$
$$y'(s) = \sin\theta\,(s)$$

Boundary conditions :

$$\theta(0) = \theta(l) = 0,$$
$$x(0) = y(0) = 0,$$
$$x(l) = l(1-\Delta), \qquad y(l) = \frac{2\,H_0}{1-\Delta}$$

Constructing the Lagrangian :

$$\mathcal{L} = \frac{L}{l}\left\{\int_{s=0}^{l} \frac{1}{2}\,\theta'^2(s) + \left[n_x(x' - \cos\theta) + n_y(y' - \sin\theta)\right] ds\right\}$$

$$\frac{\delta\mathcal{L}}{\delta(\cdot)}$$

Differential equation system :

$$x'(s) = \cos\theta\,(s),$$
$$y'(s) = \sin\theta\,(s),$$
$$\theta''(s) = n_x \sin\theta(s) - n_y\cos\theta(s)$$

$$\frac{\partial\mathcal{L}}{\partial l} \Rightarrow \frac{1}{2}\theta'^2(l) + n_x\cdot\Delta = \frac{n_y}{2}\frac{y(l)}{l}$$

Unknown & Variables :

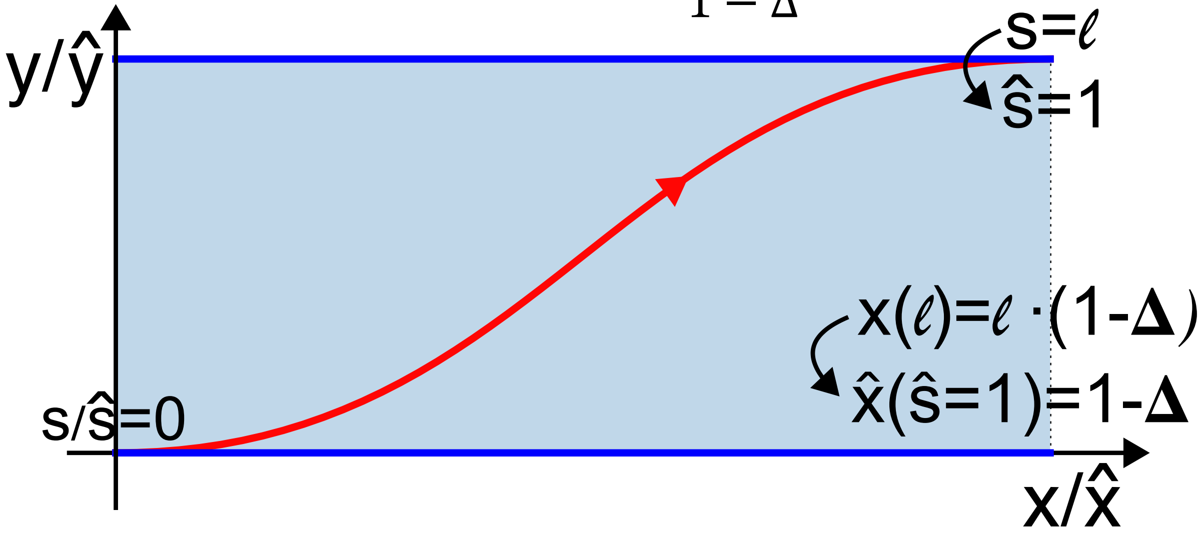
$$l\,,\qquad \theta(s),\qquad x(s),\qquad y(s)$$

Differential equation system :

$$x'(s) = cos\,\theta\,(s),$$
$$y'(s) = sin\,\theta\,(s),$$
$$\theta''(s) = n_x\,sin\,\theta(s) - n_y\,cos\,\theta(s)$$
$$\frac{1}{2}\theta'^2(l) + n_x\cdot\Delta = \frac{n_y}{2}\frac{y(l)}{l}$$

Boundary conditions :

$$\theta(0) = \theta(l) = 0,\qquad x(0) = y(0) = 0,$$
$$x(l) = l(1-\Delta),\qquad y(l) = \frac{2\,H_0}{1-\Delta}$$



This is a Self-similar system
Introducing variables :

$$\hat{s} = \frac{s}{l}, \hat{x} = \frac{x}{l}, \hat{y} = \frac{y}{l},$$

$$\hat{n}_x = \frac{n_x}{l^2}, \hat{n}_y = \frac{n_y}{l^2}$$

Unknown & Variables :

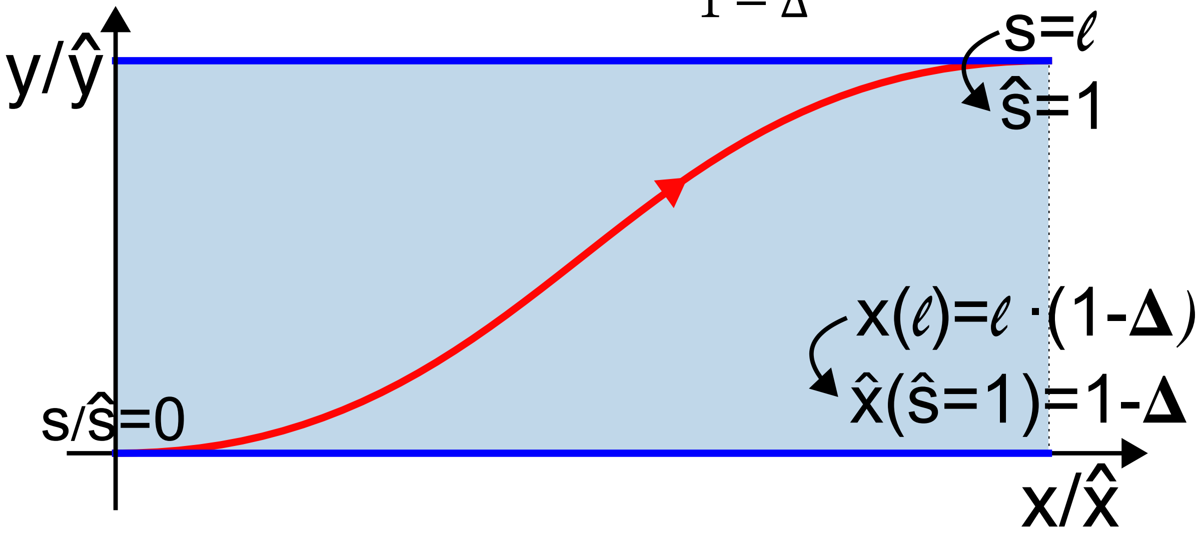
$$l\,,\qquad \theta(s),\qquad x(s),\qquad y(s)$$

Differential equation system :

$$x'(s) = \cos\theta\,(s),$$
$$y'(s) = \sin\theta\,(s),$$
$$\theta''(s) = n_x \sin\theta(s) - n_y \cos\theta(s)$$
$$\frac{1}{2}\theta'^2(l) + n_x\cdot\Delta = \frac{n_y}{2}\frac{y(l)}{l}$$

Boundary conditions :

$$\theta(0) = \theta(l) = 0,\qquad x(0) = y(0) = 0,$$
$$x(l) = l(1-\Delta),\qquad y(l) = \frac{2\,H_0}{1-\Delta}$$



$$\hat{s} = \frac{s}{l},\quad \hat{x} = \frac{x}{l},\quad \hat{y} = \frac{y}{l},\quad \hat{n}_x = \frac{n_x}{l^2},\quad \hat{n}_y = \frac{n_y}{l^2}$$

Self-similar system :

$$\hat{\theta}'(\hat{s}) = \kappa(\hat{s})$$
$$\hat{x}'(\hat{s}) = \cos\hat{\theta}\,(\hat{s})$$
$$\hat{y}'(\hat{s}) = \sin\hat{\theta}\,(\hat{s})$$
$$\hat{\theta}''(\hat{s}) = \hat{n}_x \sin\hat{\theta}(\hat{s}) - \hat{n}_y \cos\hat{\theta}(\hat{s})$$
$$\frac{1}{2}\hat{\theta}'^2(1) + \hat{n}_x\cdot\Delta = \frac{\hat{n}_y}{2}\hat{y}(1)$$

Boundary conditions :

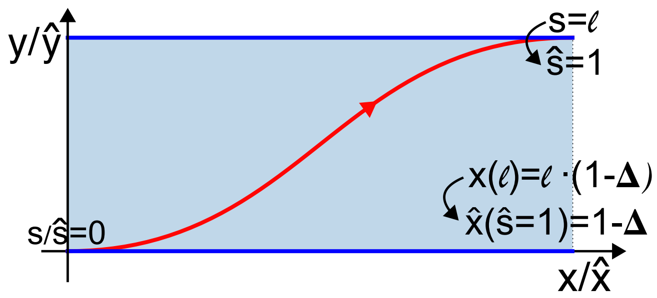
$$\hat{\theta}(0) = \hat{\theta}(1) = 0,\qquad \hat{x}(0) = \hat{y}(0) = 0$$
$$\hat{x}(1) = 1-\Delta$$

Solving self-similar system, we can calculate :

$$\hat{E} = \int_0^1 \frac{1}{2}\,\hat{\theta}'^2(\hat{s})\,d\hat{s},$$
$$\hat{y}_{end} = \hat{y}(1) = \frac{2\,H_0}{(1-\Delta)l},$$

which are functions of Δ

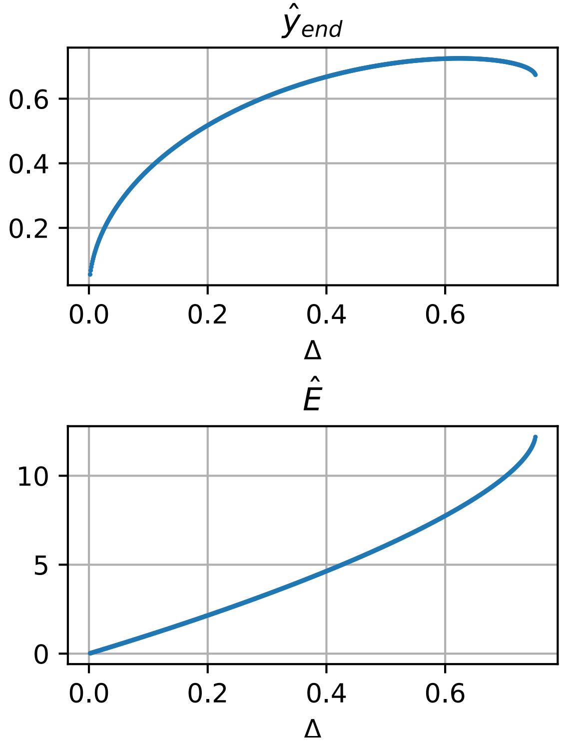
Single phase : deriving from self-similar solution



Solution of self-similar system :

$$\hat{E} = \int_0^1 \frac{1}{2}\,\hat{\theta}'^2(\hat{s})\,d\hat{s}, \qquad \hat{y}_{end} = \hat{y}(1) = \frac{2\,H_0}{(1-\Delta)l},$$

which are functions of Δ

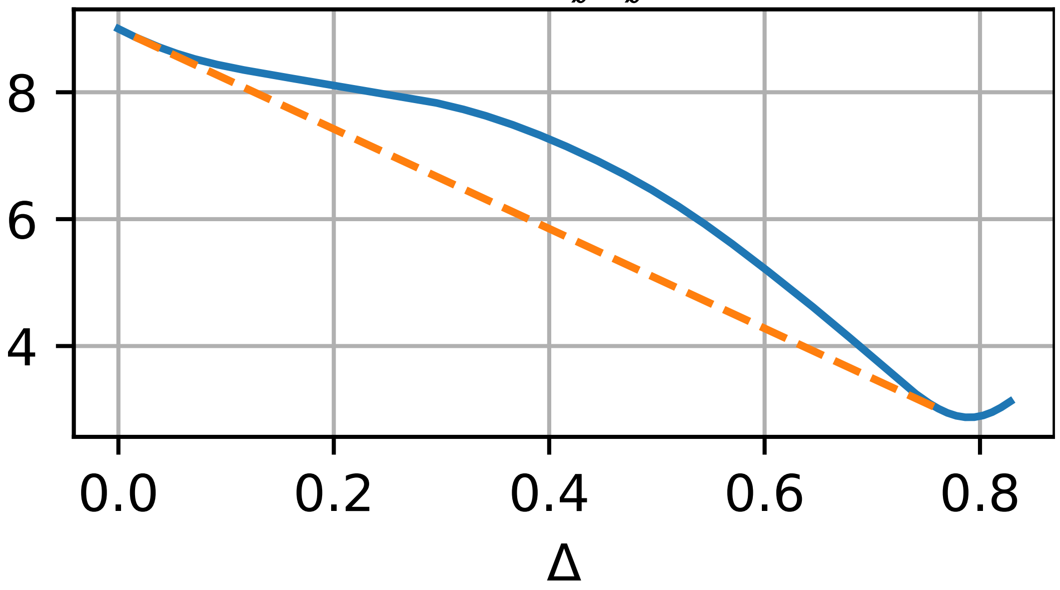


Finding the variable length l :

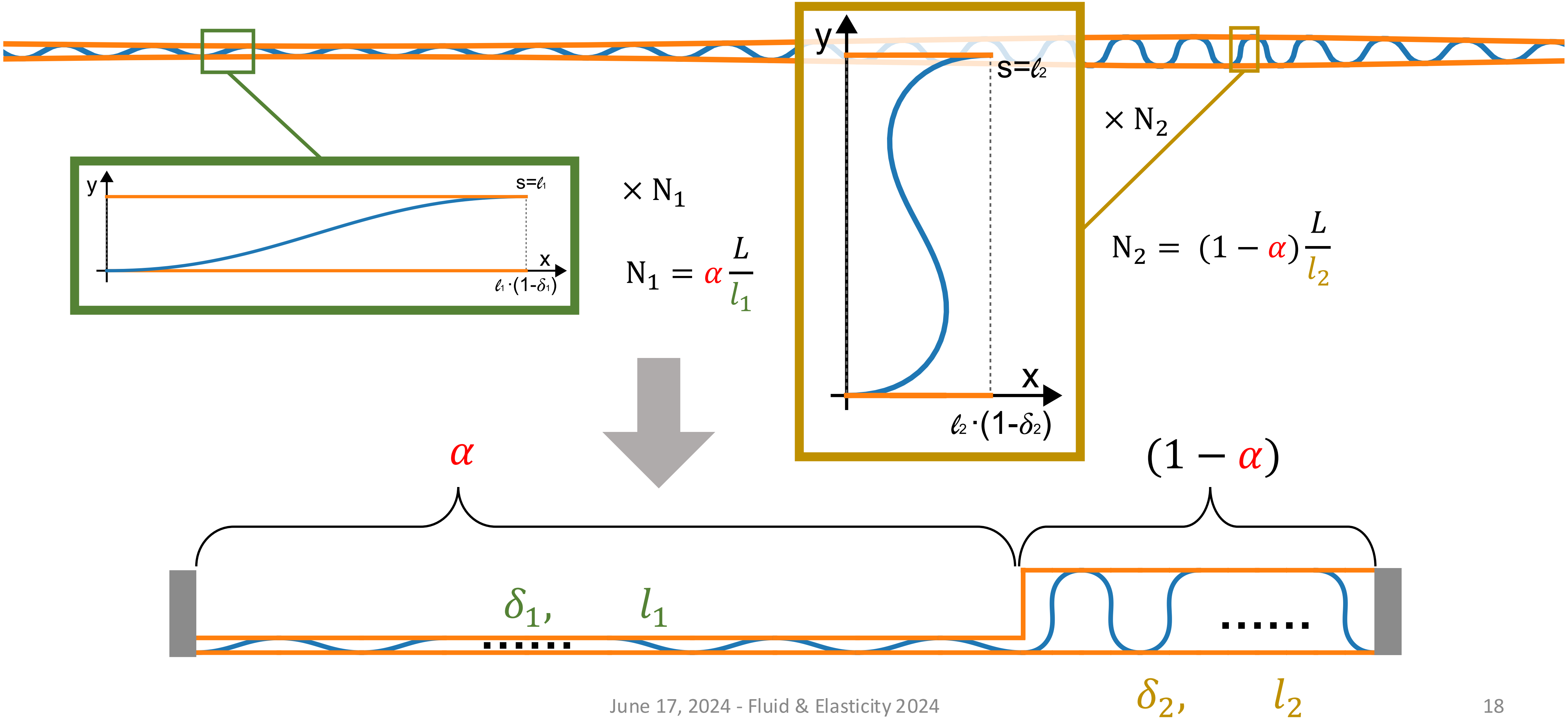
$$y(l) = \frac{2\,H_0}{1-\Delta} = l\cdot\hat{y}_{end}(\Delta)$$

$$\Longrightarrow l = \frac{2\,H_0}{(1-\Delta)\cdot\hat{y}_{end}(\Delta)}$$

$$s = l\cdot\hat{s},$$
$$x = l\cdot\hat{x},$$
$$y = l\cdot\hat{y},$$
$$E = \hat{E}/l$$



Inhomogeneous wrinkles : 2 phases



Energy :

$$\mathcal{E}_{tot} = N_1 E_1 + N_2 E_2 + 2L(1-\Delta)$$ → constant

$$N_1 = \alpha \frac{L}{l_1}, \qquad N_2 = (1-\alpha)\frac{L}{l_2}$$

For each phase :

Energy : $E_i = \frac{\hat{E}(\delta_i)}{l_i}$

Actual liquid thickness :

$$h_i = l_i \cdot \hat{y}_{end}(\delta_i)(1-\delta_i)$$

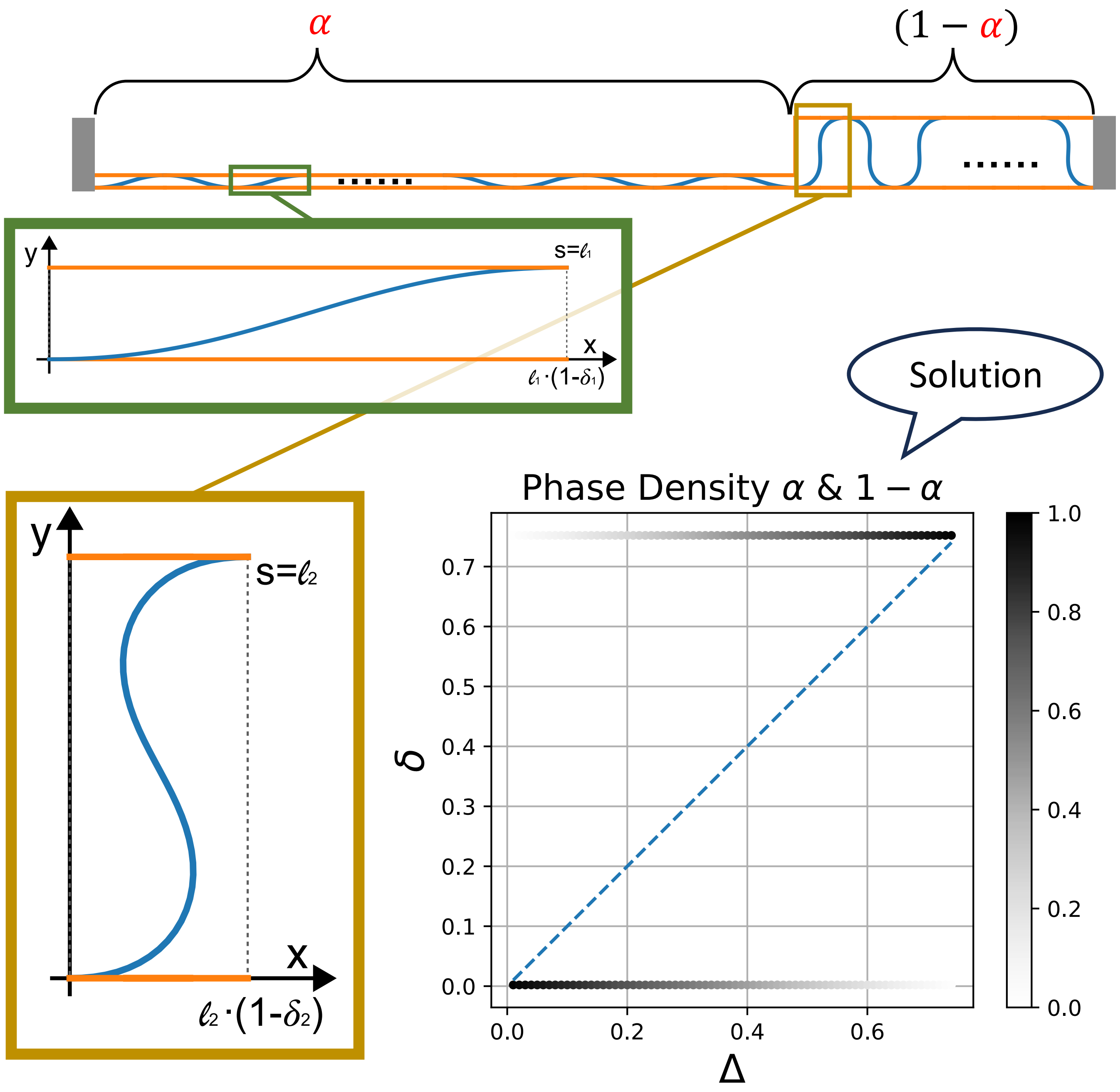
Objective :

$$\min_{(\alpha, l_i, \delta_i)} L\left[\alpha \frac{\hat{E}(\delta_1)}{l_1^2} + (1-\alpha)\frac{\hat{E}(\delta_2)}{l_2^2}\right]$$

Constraints :

Displacement control : $\alpha\delta_1 + (1-\alpha)\delta_2 = \Delta$

Fixed volume : $\alpha h_1 + (1-\alpha) h_2 = H_0$



Thank you for listening !

Questions ? Remarks ?